

Here are two practice problems that target common mistakes from the first exam. Try to work through them the same way you would on an exam (without your notes, but with the formula sheet I gave you). Take as much time as you need, so you can focus on the content and not the pressure that comes from being ‘on the clock.’ Email me or stop by my office if you get stuck. Then, when you are done, come show me your work and we can talk about any questions you have.

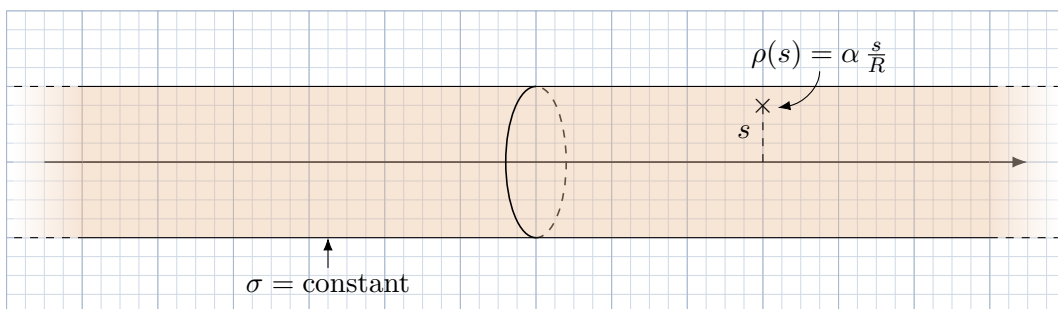
### Problem 1: Gauss’s Law

A very long cylinder with radius  $R$  has charge distributed throughout its volume and on its surface. The volume charge density changes depending on the distance  $s$  from the axis of the cylinder as

$$\rho = \alpha \frac{s}{R}, \quad (1)$$

where  $\alpha$  is just a constant. The charge on the surface is uniform, so the surface charge density  $\sigma$  is constant.

- Use Gauss’s Law to find the electric field inside ( $s < R$ ) and outside ( $s > R$ ) the cylinder. Be clear about your choice of Gaussian surfaces, the flux through these surfaces, and how much charge they contain.
- Show that the electric field has the correct discontinuity at the surface of the cylinder ( $s = R$ ).



### Problem 2: Two Charged Hemispheres

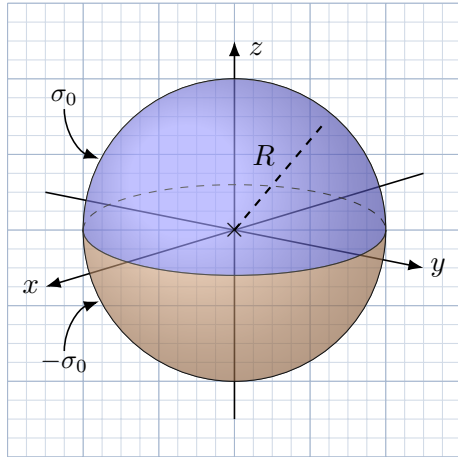
Charge is spread out over the surface of a sphere of radius  $R$ . The surface charge density is

$$\sigma = \begin{cases} +\sigma_0 & 0 \leq \theta \leq \frac{\pi}{2} \\ -\sigma_0 & \frac{\pi}{2} < \theta \leq \pi \end{cases}, \quad (2)$$

where  $\sigma_0$  is a constant and  $\theta$  is the usual polar angle in spherical polar coordinates. In other words, the surface charge density is a positive constant on the top hemisphere, and a negative constant on the bottom hemisphere.

- What is the total charge on the sphere?
- Find the potential at a point outside the sphere on the  $z$  axis, above the north pole (i.e., with  $z > R$ ).
- Use your answer to the last part to find the electric field at the same point.

In parts (b) and (c), be sure to clearly set up your calculation by identifying things like  $\vec{r}$ ,  $\vec{r}'$ , and  $\vec{z}$  before writing out the integral you want to evaluate.



The charge distribution from Problem 2: there is a positive surface charge density  $\sigma_0$  on the top half of the sphere, and a negative surface charge density  $-\sigma_0$  on the bottom half.