Here are two practice problems that target common mistakes from the first exam. Try to work through them the same way you would on an exam (without your notes, but with the formula sheet I gave you). Take as much time as you need, so you can focus on the content and not the pressure that comes from being on the clock. Email me if you get stuck. Then, when you are done, upload your work to your Sakai dropbox and we can talk about any questions you have.

Problem 1: Gauss's Law

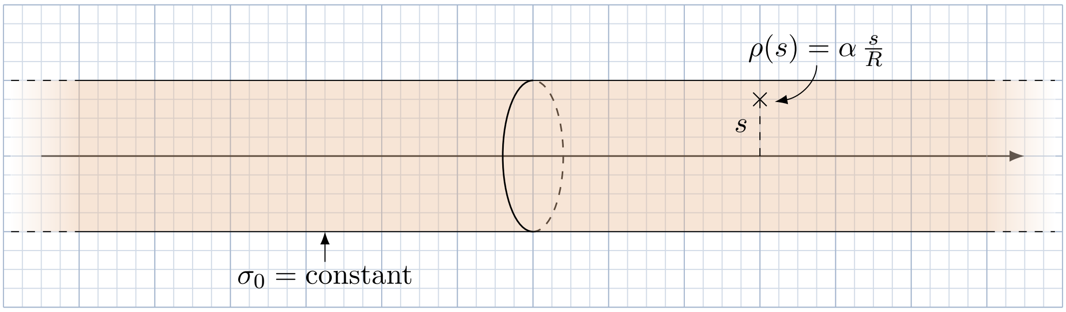
A very long cylinder with radius R has charge distributed throughout its volume and on its surface. The volume charge density changes depending on the distance s from the axis of the cylinder as

$$\rho = \rho_0 \, \frac{s}{R} \, , \tag{1}$$

where ρ_0 is just a constant. The charge on the surface is uniform – the surface charge density σ_0 is constant.

(a) Use Gauss's Law to find the electric field inside ($s < R$) and outside ($s > R$) the cylinder. Be clear about your choice of Gaussian surfaces, the flux through these surfaces, and how much charge they contain.

(b) Show that the electric field has the correct discontinuity at the surface of the cylinder ($s = R$).



Problem 2: Two Charged Hemispheres

Charge is spread out over the surface of a sphere of radius R, with positive charge on the upper hemisphere and negative charge on the lower hemisphere according to

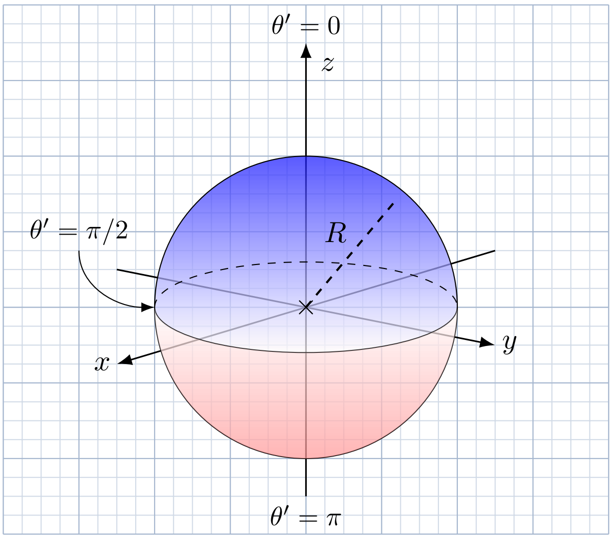
$$\sigma(\theta') = \sigma_0 \, \cos\theta' \tag{2}$$

Here σ_0 is a constant and θ' is the usual polar angle (in spherical polar coordinates) for the bit of charge located at $\vec{r}'$. The sphere is hollow on the inside – all of the charge is on the surface.

(a) Find the electrostatic potential $V(0, 0, z)$ at a point on the z axis. You can assume that $z > 0$, but the point can be either outside ($z > R$, so $\sqrt{(R-z)^2} = z - R$) or inside ($0 < z < R$, so $\sqrt{(R-z)^2} = R - z$) the sphere.

(b) Use $V(0, 0, z)$ to find the electric field $\vec{E}(0, 0, z)$ at points on the positive z-axis.

In part (a), be sure to clearly set up your calculation by identifying things like $\vec{r}$, $\vec{r}'$, and $\vec{\mathscr{r}}$ before writing out the integral you want to evaluate.



Problem 2: The surface charge density $\sigma(\theta') = \sigma_0 \cos\theta'$ is positive on the upper hemisphere ($0 < \theta' < \pi/2$), zero at the equator ($\theta' = \pi/2$), and negative on the lower hemisphere ($\pi/2 < \theta' < \pi$).