ANOTHER INTEGRAL FROM HW 2

- In problems 18, 4 you encounter two sorts of integrals, of the form

\[ \int_{-A}^{A} \frac{(x-x')}{((x-x')^{2} + \alpha^{2})^{3/2}} \text{ and } \int_{-A}^{A} \frac{(y+y')}{((x-x')^{2} + y^{2} + z^{2})^{3/2}} \]

The 1st can be done w/ the substitution \( u = (x-x')^{2} + y^{2} + z^{2} \).
But the 2nd is trickier. Forgetting the factor in the numerator, which does not depend on \( x' \), we need to evaluate something like:

\[ \int_{-A}^{A} \frac{1}{((x-x')^{2} + \alpha^{2})^{3/2}} \text{ w/ } \alpha^{2} = y^{2} + z^{2} \text{ in this case.} \]

Consider the substitution \( x-x' = \alpha \tan u \).

\( x-x' = \alpha \tan u \Rightarrow -dx' = \alpha \sec^{2}u \ du \)

\( (x-x')^{2} + \alpha^{2} = \alpha^{2} \tan^{2} u + 1 = \alpha^{2} \sec^{2} u \)

\[ \int_{-A}^{A} \frac{1}{((x-x')^{2} + \alpha^{2})^{3/2}} = \int_{-A}^{A} du / \alpha \sec^{2}u (1 - 1/\alpha^{2} \sec^{2}u) \]

\[ = -\frac{1}{\alpha^{2}} \int du \cos u = -\frac{1}{\alpha^{2}} \sin u \]

\[ = - \frac{1}{\alpha^{2}} \sin \left( \tan^{-1} \left( \frac{x-x'}{\alpha} \right) \right) \]

\[ \Rightarrow \int_{-A}^{A} \frac{1}{((x-x')^{2} + \alpha^{2})^{3/2}} = -\frac{1}{\alpha^{2} \sqrt{(x-x')^{2} + \alpha^{2}}} \]