Homework 5: $\mathbb{C}$ Fourier Series and Periods Other Than 2π
Due Wednesday, February 21

This assignment involves lots of integrals. For problems 1-4 you must evaluate the integrals by hand and show all your work to receive credit. On problem 5 you may use MATHEMATICA, but you must attach a printout of the notebook containing your calculations to receive full credit. The results you use must be readable and easy for the grader to find. You **may not** use any other software or online resource for this assignment.

Problem 1: A $\mathbb{C}$ Fourier Series

Find the complex Fourier Series representation of the square wave with period 2π described by

$$f(x) = \begin{cases} -1\,, & -\pi \leq x < 0 \\ 1\,, & 0 \leq x < \pi\,. \end{cases} \tag{1}$$

Write the general form of the series (with the exact form of the coefficients a_n and b_n for arbitrary n), and then write out the first three non-zero terms in the series.

Problem 2: Another $\mathbb{C}$ Fourier Series

Find the complex Fourier Series representation of the function

$$f(x) = |x| \tag{2}$$

on $-\pi \leq x < \pi$. Write the general form of the series (with the exact form of the coefficients a_n and b_n for arbitrary n), and then write out the first three non-zero terms in the series.

Problem 3: Fourier Series, Parseval's Theorem, and Sums

Find the Fourier Series representation (using sines and cosines) of the function

$$f(x) = \begin{cases} 0\,, & 0 \leq x < 1 \\ 1\,, & 1 \leq x < 2\,. \end{cases} \tag{3}$$

Give the exact form of the coefficients a_n and b_n (for arbitrary n). Now use your results along with Parseval's Theorem to evaluate the sum

$$\sum_{n=0}^{\infty} \frac{1}{(2k+1)^2}\,. \tag{4}$$

You must show how you obtain your answer for the last part; if you simply write down the value of the sum (which can be easily evaluated with Mathematica, which you aren't supposed to be using on this problem) you will not receive credit.

Problem 4: There Sure Are a Lot of Fourier Series

Find the Fourier Series representation (using sines and cosines) of the function

$$f(x) = (x-2)^2 \tag{5}$$

on the interval $0 \le x < 4$. Write the general form of the series (with the exact form of the coefficients a_n and b_n for arbitrary n), and then write out the values of the coefficients for $n = 1, 2, 3$.

Problem 5: A Plucked String

In class we used the example of a plucked string to motivate Fourier Series. Now we've learned enough to work out the result I quoted in that first lecture. The shape of the string at $t = 0$, before it was released, was described by

$$y(x,0) = \begin{cases} A\,\frac{2}{L}\,x\,, \;\; 0 \le x < \frac{L}{2} \\ A\,\frac{2}{L}\,(L-x)\,, \; \frac{L}{2} \le x < L\,. \end{cases} \tag{6}$$

Find the odd extension of this shape to $0 \le x \le 2L$, and then determine the coefficients of its Fourier sine series representation.