Homework 4: Fourier Series

Due Monday, February 12

This assignment involves lots of integrals. You may use MATHEMATICA to check your results, but you must evaluate the integrals by hand and show all your work to receive credit. You may also use an integral table, as long as it is a) an actual book, and b) you properly cite it (name, authors, page number, formula number). You **may not** use any online resource for this assignment.

Problem 1: Essential Integrals

Assuming n and m are non-negative integers, perform the following integrals to confirm the results we gave in class:

$$\int_{-\pi}^{\pi} dx\, \sin(nx)\, \sin(mx) = \begin{cases} 0\,, & m \neq n \\ \pi\,, & m = n \end{cases} \tag{1}$$

$$\int_{-\pi}^{\pi} dx\, \cos(nx)\, \cos(mx) = \begin{cases} 0\,, & m \neq n \\ \pi\,, & m = n \end{cases} \tag{2}$$

$$\int_{-\pi}^{\pi} dx\, \sin(nx)\, \cos(mx) = 0\,. \tag{3}$$

HINT: There is more than one way to evaluate these integrals. First, you can use the multi-angle formulas

$$\sin(nx \pm mx) = \sin(nx)\, \cos(mx) \pm \sin(mx) \cos(nx) \tag{4}$$

$$\cos(nx \pm mx) = \cos(nx)\, \cos(mx) \mp \sin(mx) \sin(nx) \tag{5}$$

to write the integrands as the sum or difference of two trig functions. Alternately, if you've seen Euler's formula you can write the trig functions using exponentials and complex numbers.

$$\sin(nx) = \frac{1}{2\,i}\left(e^{inx} - e^{-inx}\right) \qquad \cos(nx) = \frac{1}{2}\left(e^{inx} + e^{-inx}\right)\,. \tag{6}$$

You can then "f.o.i.l." the product of the two trig functions to get an integrand that just involves exponentials.

Whichever approach you use, be careful to distinguish between the cases $m = n$ and $m \neq n$. When you evaluate integrals you will get factors like $1/(n - m)$, which are harmless when $m \neq n$ but don't make sense for $n = m$.

Problem 2: A Fourier Series

Find the Fourier Series representation of the function

$$f(x) = \begin{cases} 0\,, & -\pi \leq x \leq 0 \\ x\,, & 0 \leq x \leq \pi\,. \end{cases} \tag{7}$$

Give the exact form of the coefficients a_n and b_n (for arbitrary n) and then write out their actual values for $n = 0, 1, 2, 3$. (Here, $n = 0$ refers to the a_0 term in the series.)

Problem 3: Another Fourier Series

Find the Fourier Series representation of the function

$$f(x) = \begin{cases} -1\,, & -\pi \le x \le 0 \\ \;\;1\,, & \;\;0 \le x \le \pi\ . \end{cases} \tag{8}$$

Give the exact form of the coefficients a_n and b_n (for arbitrary n) and then write out their actual values for $n = 0, 1, 2, 3$. (Here, $n = 0$ refers to the a_0 term in the series.)

Problem 4: Wow we sure are doing a lot of Fourier Series

Find the Fourier Series representation of the function

$$f(x) = \begin{cases} 0\,, & -\pi \le x \le 0 \\ \cos(x)\,, & \;\;0 \le x \le \pi\ . \end{cases} \tag{9}$$

Give the exact form of the coefficients a_n and b_n (for arbitrary n) and then write out their actual values for $n = 0, 1, 2, 3$. (Here, $n = 0$ refers to the a_0 term in the series.)