Homework 4: Electric Potential
Due: Thursday, September 21

Problem 1: An Impossible Electric Field

One of these vector fields cannot possibly be an electric field. Which one?

(a) $\vec{E}_1 = \alpha\,(z^2 + 2\,x\,y)\,\hat{x} + \alpha\,x^2\,\hat{y} + 2\,\alpha\,z\,x\,\hat{z}$

(b) $\vec{E}_2 = \alpha\,x\,y\,\hat{x} + 2\,\alpha\,y\,z\,\hat{y} + 3\,\alpha\,x\,z\,\hat{z}$

Here α is a constant with appropriate units. For the $\vec{E}$ which could be an electric field, find the potential at a point with coordinates (x, y, z). Use the *origin* as the reference point and find V by integrating $d\vec{\ell} \cdot \vec{E}$ along any path that starts at the origin and ends at the point (x, y, z). Check that $-\vec{\nabla}V$ for your answer gives the correct $\vec{E}$.

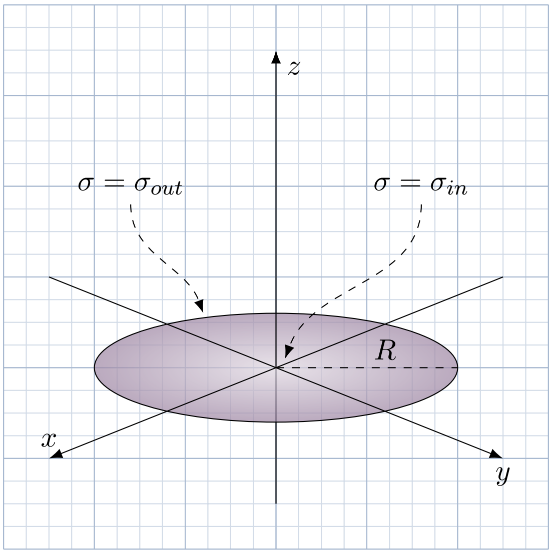
> HINT: You can use any path between the origin and (x, y, z) to find V – they will all give the same result – but to perform the integral you must choose *some* path. Use one that makes the integral easy. For instance, integrate along the x-axis from $(0, 0, 0)$ to $(x, 0, 0)$, then in the y-direction from $(x, 0, 0)$ to $(x, y, 0)$, and so on.

Problem 2: Non-Uniform Surface Charge on a Disk

Find the electric potential at a point above the center of a disk of radius R with a surface charge density that changes linearly from the center of the disk to its outer edge:

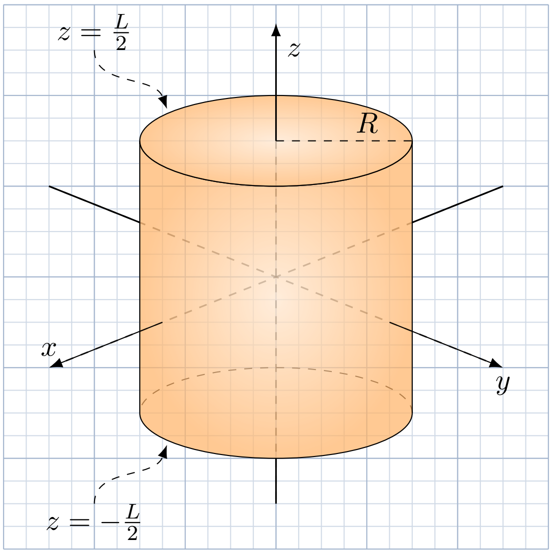
$$\sigma(s) = \sigma_{in} + \frac{s}{R}\left(\sigma_{out} - \sigma_{in}\right),$$

where σ_{in} and σ_{out} are constants, and s is the distance from the center of the disk. To make things easy, assume that the disk sits in the x-y plane, with the origin at the center of the disk, and use cylindrical polar coordinates. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution.



Problem 3: Potential due to a Uniform Charged Cylinder

A solid cylinder of length L and radius R carries a uniform volume charge density ρ. Find the potential at a point on the axis of the cylinder, a distance z from its center. Assume that $z > L/2$ so the point is outside of the cylinder.



Problem 4: Potential due to a Solid Uniform Sphere

You already know that the electric potential outside a uniform charged solid sphere of radius R and total charge q is the same as a point charge. Calculate the electric potential inside the sphere using the formula

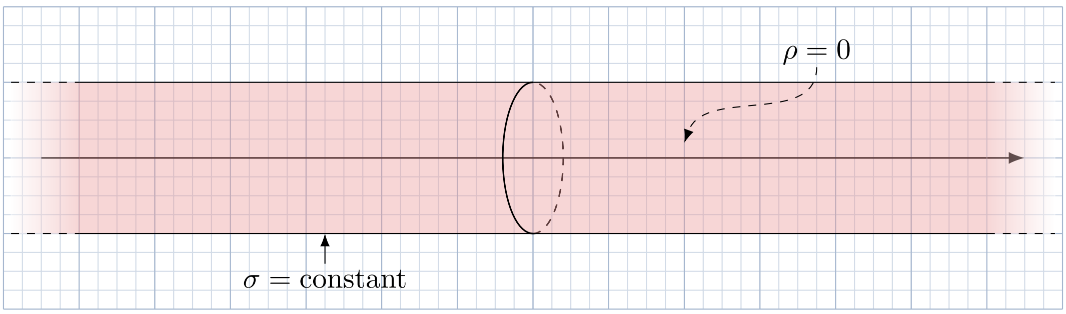
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} d\tau' \, \frac{\rho(\vec{r}')}{\mathscr{r}} \, .$$

You can check your answer by taking the $R_i \to 0$ limit of the electric potential for a spherical shell of inner radius R_i and outer radius R, which we will work out in class. Finally, use the gradient of your answer to find the electric field inside and outside the sphere.

Problem 5: Gauss' Law for a Long, Hollow Cylinder

Use Gauss' law to find the electric field inside and outside a long, hollow, cylindrical tube of radius R which carries a uniform surface charge σ. Check that your result agrees with the expected discontinuity in the component of the electric field normal to the surface:

$$\left(\vec{E}_{out} - \vec{E}_{in}\right)\Big|_{\text{surface}} = \frac{\sigma}{\epsilon_0}\,\hat{n}$$



Problem 6: Potential due to a Charged Semicircle

A semicircle of radius R and uniform line charge λ sits in the x-y plane. The equation describing the semicircle is

$$x^2 + y^2 = R^2 \quad \text{with} \quad x \geq 0 \ ,$$

so that the center of the full circle sits at the origin. Calculate the potential at a point on the z axis using the integral

$$V = \frac{1}{4\pi\epsilon_0} \int d\ell' \, \frac{\lambda(\vec{r}\,')}{\mathscr{r}} \ .$$

(This is a pretty simple calculation — if it looks complicated you should double-check your expressions for $\vec{r}$, $\vec{r}\,'$, $\vec{\mathscr{r}}$, and $|\vec{\mathscr{r}}|$.) The electric field at a point on the z axis should have both y and z components. Can you determine the y and z components at $(0, 0, z)$ using your expression for the potential? Why or why not?