

Homework 4: Electric Potential

Due: Friday, September 23

Problem 1: An Impossible Electric Field

One of these vector fields cannot possibly be an electric field. Which one?

(a) $\vec{E} = \alpha y z \hat{x} + \alpha z (x - 4y) \hat{y} + \alpha y (x - 2y) \hat{z}$.

(b) $\vec{E} = -3\alpha x z \hat{x} + \alpha x y \hat{y} + 2\alpha y z \hat{z}$

Here α is a constant with appropriate units. For the \vec{E} which could be an electric field, find the potential at a point with coordinates (x, y, z) . Use the *origin* as the reference point and find V by integrating $d\vec{\ell} \cdot \vec{E}$ along some path from the origin to the point. Check that $-\vec{\nabla}V$ for your answer gives the correct \vec{E} .

Hint: You can use any path between the origin and (x, y, z) to find V – they will all give the same result – but to perform the integral you must choose *some* path. Use one that makes the integral easy. For instance, along the x -axis from $(0, 0, 0)$ to $(x, 0, 0)$, then in the y -direction from $(x, 0, 0)$ to $(x, y, 0)$, and so on.

Problem 2: Non-Uniform Surface Charge on a Disk

A disk with radius R sits in the x - y plane, with its center at the origin. Find the electric potential at a point on the z -axis for a charge distributions that changes linearly from the center of the disk to its outer edge:

$$\sigma(s) = \sigma_{in} + \frac{s}{R} (\sigma_{out} - \sigma_{in}),$$

where σ_{in} and σ_{out} are constants, and s is the distance from the z -axis. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution!

Problem 3: Potential due to a Uniform Charged Cylinder

A solid cylinder of length L and radius R carries a uniform volume charge density ρ . Find the potential at a point on the axis of the cylinder, *outside* the cylinder, at a distance z from its center. (Since the point is outside, you are assuming $z > L/2$.)

Problem 4: Potential due to a Solid Uniform Sphere

Calculate the electric potential inside a uniform charged solid sphere of radius R and total charge q using the formula

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} d\tau' \frac{\rho(\vec{r}')}{z}.$$

You can check your answer by taking the $R_i \rightarrow 0$ limit of the electric potential for a spherical shell of inner radius R_i and outer radius R , which we will work out in class. Finally, take (minus) the gradient of your answer and verify that you get the correct expression for the electric field inside and outside a uniformly charged sphere.

Problem 5: Gauss' Law for a Long, Hollow Cylinder

Use Gauss' law to find the electric field inside and outside a long, hollow, cylindrical tube of radius R which carries a uniform surface charge density σ . Check that your result is consistent with the expected discontinuity in the component normal to the surface:

$$\left(\vec{E}_{out} - \vec{E}_{in} \right) \Big|_{\text{surface}} = \frac{\sigma}{\epsilon_0} \hat{n}$$

Problem 6: Potential due to a Charged Semicircle

A semicircle of radius R and uniform line charge λ sits in the x - y plane. The equation describing the semicircle is

$$x^2 + y^2 = R^2 \quad \text{with} \quad y \geq 0 ,$$

so that the 'center' of the full circle sits at the origin. Calculate the potential at a point on the z axis using the integral

$$V = \frac{1}{4\pi\epsilon_0} \int d\ell' \frac{\lambda(\vec{r}')}{z} .$$

(This is a pretty simple calculation — if it looks complicated you should double-check your expressions for \vec{r} , \vec{r}' , \vec{z} , and $|\vec{z}|$.) The electric field at a point on the z axis should have both y and z components. Can you determine the y and z components at $(0, 0, z)$ using your expression for the potential? Why or why not?