Homework 3: Electrostatics
Due Thursday, September 14

Important: You may not use MATHEMATICA or similar software on this assignment. This includes Maple, Matlab, Wolfram Alpha, or any other system that performs integrals for you. Show enough work to convince me that you have worked out the integrals.

Problem 1: Electric field due to charged square sheet

Find the electric field at a height z directly above ($z > 0$) the center of a square sheet with sides of length L and a uniform surface charge σ. Check your result for the limiting cases $L \to \infty$ and $z \gg L$.

> HINT: When I say "check your result", I mean that your answer should reduce to something more familiar in those two cases. When $L \to \infty$ it should look like the electric field for an infinite charged sheet. When $z \gg L$ it should look (approximately) like the electric field due to a point charge $q = L^2\sigma$.

Problem 2: Charge density for a point charge

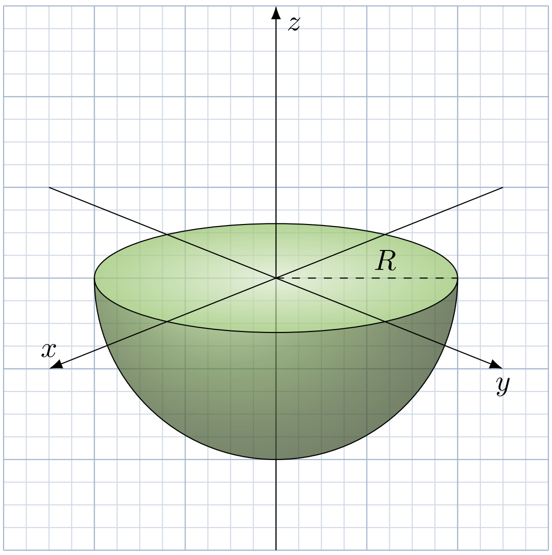
Suppose there is a point charge q located at $\vec{r}\,'$. Determine the volume charge density $\rho(\vec{r})$ by applying the differential form of Gauss's law to the electric field produced by the point charge. (This result showed up in our discussion of Gauss's Law in class – check your notes if you have questions.)

Problem 3: Electric field due to a charged hemisphere

A hemisphere with radius R has a constant volume charge density ρ. Find the electric field at a point on the hemisphere's axis of symmetry, on the side where the hemisphere is flat. To be a little more specific, let's set up our coordinates so the hemisphere is

$$x^2 + y^2 + z^2 \leq R^2 \quad \text{with} \quad z \leq 0\ . \tag{1}$$

Find the electric field at a point on the z-axis with $z > 0$.



> HINT: Use a combination of spherical polar coordinates and Cartesian unit vectors to set up and evaluate the integral.

Problem 4: Electric field due to a charged line segment

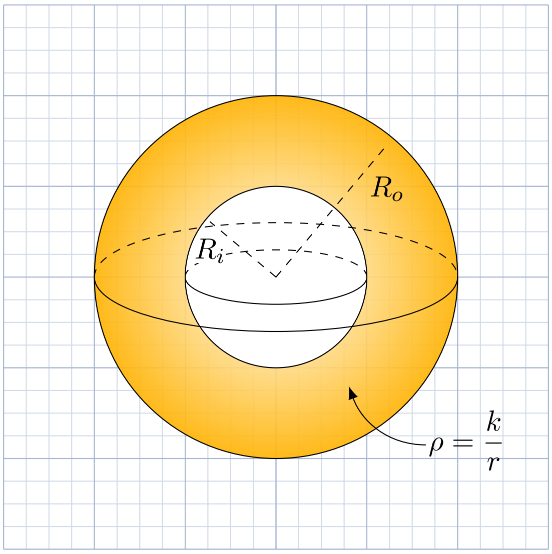
A straight line segment $-L/2 \leq y \leq L/2$ carries a uniform line charge λ. Find the electric field at an arbitrary point $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$. What happens to your result if you let $L \to \infty$, while keeping λ constant?

Problem 5: Electric field due to charged spherical shell

A hollow spherical shell with inner radius R_i and outer radius R_o carries a volume charge density

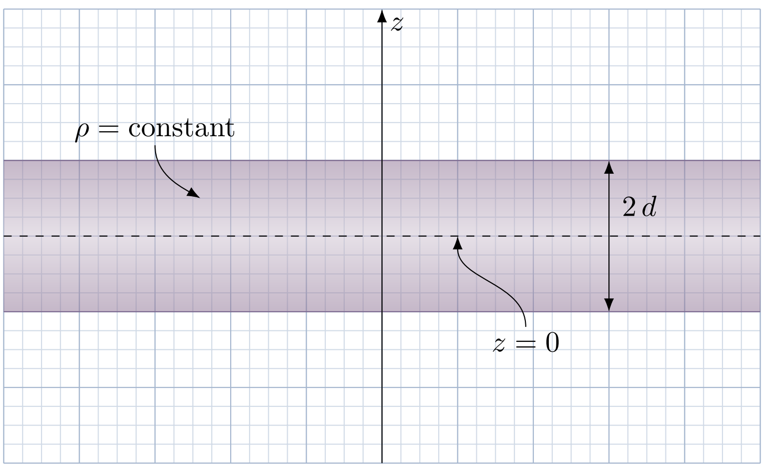
$$\rho = \frac{k}{r} \qquad (k \text{ is a positive constant}) \tag{2}$$

in the region $R_i \leq r \leq R_o$. Use Gauss's law to find the electric field in the three regions (i) $r < R_i$, (ii) $R_i \leq r \leq R_0$, (iii) $r > R_o$, and then draw a plot of $|\vec{E}|$ as a function of r.



Problem 6: Electric field due to an infinite plane slab

An infinite plane slab of thickness $2d$ carries a uniform volume charge density ρ. Find the electric field as a function of z, where $z = 0$ at the center of the slab. Plot the electric field versus z, treating it as positive when it points in the $\hat{z}$ direction and negative when it points in the $-\hat{z}$ direction. (You can assume that the charge density ρ is positive.)



Problem 7: Gauss's Law

Through various measurements you determine that the electric field in a region is given (in cylindrical polar coordinates) by

$$\vec{E} = a\,\frac{s^4}{R^4}\,\hat{s} + b\,\frac{R}{s}\,\hat{\phi} + c\,\frac{z^3}{R\,L^2}\,\hat{z} \tag{3}$$

where R and L are constants with units of length, and a, b, and c are all constants with appropriate units. What is the charge density ρ in this region? How much charge is contained in the cylinder $0 \leq s \leq R$, $0 \leq z \leq L$, $0 \leq \phi < 2\pi$?