Homework 3: Electrostatics
Due Thursday, September 15

Important: You may not use MATHEMATICA or similar software on this assignment. This includes Maple, Matlab, Wolfram Alpha, or *anything* that performs integrals for you! Show enough work to convince me that you have worked out the integrals.

Problem 1: Electric Field due to charged square sheet

Find the electric field at a height z above the center of a square sheet, with sides of length a, carrying a uniform surface charge σ. Check your results for the limiting cases $a \to \infty$ and $z \gg a$.

> *Hint:* When I say "check your results", I mean that your answer should reduce to something more familiar in those two cases. When $a \to \infty$ it should look like the electric field for an infinite charged sheet. When $z \gg a$ it should look (approximately) like the electric field due to a point charge $q = a^2\sigma$.

Problem 2: Charge density for a point charge

Suppose there is a point charge q located at $\vec{r}\,'$. Determine the volume charge density $\rho(\vec{r})$ by applying the differential form of Gauss' law to the point charge's electric field. (This result showed up in our discussion of Gauss' Law in class – check your notes if you have questions.)

Problem 3: Electric Field due to a charged hemisphere

A hemisphere with radius R has a constant volume charge density ρ. Find the electric field at a point on the hemisphere's axis of symmetry, on the side where the hemisphere is flat. To be a little more specific, let's set up our coordinates so the hemisphere is

$$x^2 + y^2 + z^2 \leq R^2 \quad \text{with} \quad z \leq 0 \, . \tag{1}$$

That is, the hemisphere is *below* the x-y plane. Now find the electric field at a point on the z-axis with $z > 0$.

> *Hint:* Use a combination of spherical polar coordinates and Cartesian unit vectors to set up and evaluate the integral.

Problem 4: Electric Field due to a charged line segment

A straight line segment $-L \leq x' \leq L$ carries a uniform line charge density λ. Find the electric field at an arbitrary point $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$. What happens to your result if you let $L \to \infty$, while keeping λ constant?

Problem 5: Gauss' Law

Through various measurements you determine that the electric field in a region is given (in cylindrical polar coordinates) by

$$\vec{E} = A\,\frac{s^4}{R^4}\,\hat{s} + B\,\frac{R}{s}\,\hat{\phi} + A\,\frac{z^3}{R\,L^2}\,\hat{z} \tag{2}$$

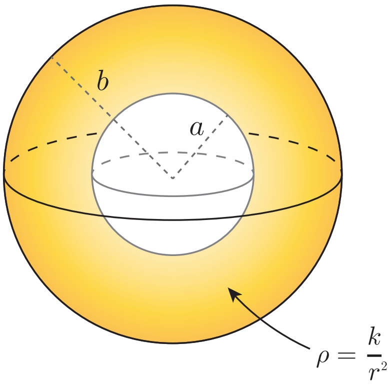
where A, B, R, and L are all constants. (If you think about how this is written, you can see that R and L have units of length, and A and B have units of N/C like the electric field.) What is the charge density in this region? Suppose the region is the cylinder $0 \leq s \leq R$, $0 \leq z \leq L$, $0 \leq \phi < 2\pi$. How much charge is contained in the cylinder?

Problem 6: Electric Field due to charged spherical shell

A hollow spherical shell with inner radius a and outer radius b carries a volume charge density

$$\rho = \frac{k}{r^2} \qquad (k \text{ is a constant}) \tag{3}$$

in the region $a \leq r \leq b$ (see the figure below). Use Gauss' law to find the electric field in the three regions (i) $r < a$, (ii) $a \leq r \leq b$, (iii) $r > b$, and then plot $|\vec{E}|$ as a function of r.



Problem 7: Electric Field due to an infinite plane slab

An infinite plane slab of thickness $2d$ carries a uniform volume charge density ρ. Find the electric field, as a function of z, where $z = 0$ at the center. Plot the electric field versus z, treating it as positive when it points in the $\hat{z}$ direction and negative when it points in the $-\hat{z}$ direction.