Homework 12: The Vector Potential, Magnetization

Due Friday, December 2

Problem 1: A rotating charged sphere

The vector potential for a spherical shell with uniform surface charge density σ, spinning with constant angular velocity $\vec{\omega}$, is derived in Example 5.11 of *Griffiths*. Repeat this derivation for a spinning solid sphere with uniform volume charge density ρ (and no surface charge density). Use the vector potential to determine the magnetic field both inside and outside the sphere.

Problem 2: An infinite solenoid

Use Ampère's law to find the magnetic field inside and outside a very long cylindrical solenoid with radius R. Now use Stoke's theorem

$$\int_{\mathcal{S}} d\vec{a} \cdot \vec{B} = \int_{\mathcal{S}} d\vec{a} \cdot (\vec{\nabla} \times \vec{A}) = \oint_{\mathcal{P}} d\vec{\ell} \cdot \vec{A}$$

to find the vector potential both inside and outside the solenoid. Clearly indicate the various surfaces and loops that you use. (Why not just use our integral formula for $\vec{A}$? Check what happens when you try to compute $\vec{A}$ that way – it won't work! So why bother calculating $\vec{A}$ if we already know $\vec{B}$ from Ampère's law? Because the vector potential is more than just a convenient way of calculating $\vec{B}$; it has physical importance of its own.)

Problem 3: Circular loop of wire . . .

A circular loop of wire with radius R lies in the x-y plane, centered at the origin, and carries a current I (running counterclockwise, as seen from above).

(a) What is the magnetic dipole moment for the loop?

(b) What is the approximate vector potential at points far ($r \gg R$) from the loop?

(c) What is the approximate magnetic field at points far from the loop?

(d) Evaluate your answer for part (c) at a point on the z axis, and compare it to the exact expression for the magnetic field (we calculated it in class using the Biot-Savart law) for $z \gg R$.

Problem 4: Multipole expansion for part of a steady current

In class we worked out the vector potential for a straight segment of current running from $z = -L$ to $z = L$. (Remember: this only makes sense if we think of the straight segment as part of a steady current. We have to interpret the vector potential we found as that segment's contribution to the vector potential for the whole current.) Calculate the first five terms ($n = 0, 1, 2, 3, 4$) in the multipole expansion of $\vec{A}$ for this straight current. This is not a closed loop of current, so you cannot assume that the monopole term in the multipole expansion is zero.

Problem 5: A magnetized cylinder

A very long cylinder with radius R has a "frozen-in" magnetization

$$\vec{M} = k\, s^2\, \hat{\phi}\ ,$$

where k is a constant and s is the distance to the cylinder's axis (the z-axis). Find the magnetic field inside and outside the cylinder using the following two methods:

(a) Find the bound currents associated with the magnetization, then use Ampère's law to determine the magnetic field.

(b) Determine $\vec{H}$ (keep in mind that there are *no* free currents in this problem), and then use the relationship between $\vec{H}$, $\vec{B}$, and $\vec{M}$ to find the magnetic field.