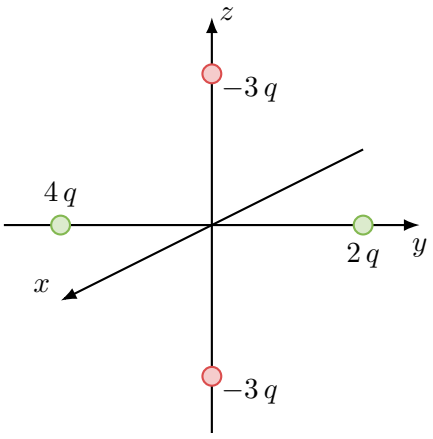
Homework 8: The Multipole Expansion

Due Monday, October 26

Problem 1: Approximate potential

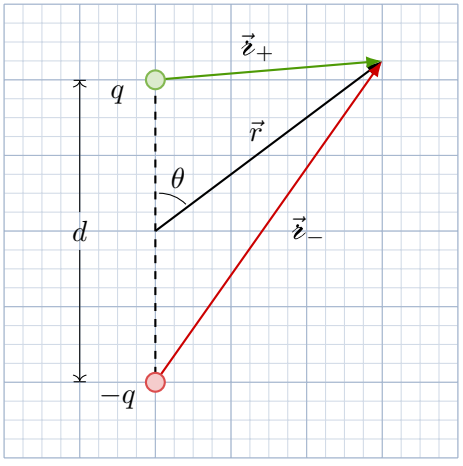
Four particles (one of charge $2q$, one of charge $4q$, and two of charge $-3q$) are placed as shown in the figure below:



Each charge is the same distance L from the origin. Find a simple approximate formula for the potential that is valid at points far from the charges ($r \gg L$). Express your answer in spherical coordinates. (By 'simple approximate formula', I mean the first term in the multipole expansion of the potential that is not zero.)

Problem 2: Quadrupole and Octopole terms for a physical dipole

Two charges, q and $-q$, are separated by a distance d as in the figure below:



The potential at a point $\vec{r}$ is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z_+} - \frac{q}{z_-}\right) . \tag{1}$$

Work out the first four terms in the Taylor expansion of the potential for $r \gg d$. Identify the monopole, dipole, quadrupole, and octopole terms in the expansion. Which ones are zero?

Problem 3: Force on a point charge due to a pure dipole

A "pure" dipole p is situated at the origin, pointing in the z direction. We worked out the potential and electric field for this dipole in class.

(a) What is the force on a point charge q located at $(0, 0, d)$ in Cartesian coordinates?

(b) What is the force on q if it is located at $(d, 0, 0)$?

(c) How much work does it take to move the charge q from $(0, 0, d)$ to $(d, 0, 0)$?

Problem 4: Multipole expansion for a line charge

A thin insulating rod, running from $z = -a$ to $z = +a$, carries a line charge $\lambda(z)$. In the following cases, find the leading term in the multipole expansion of the potential.

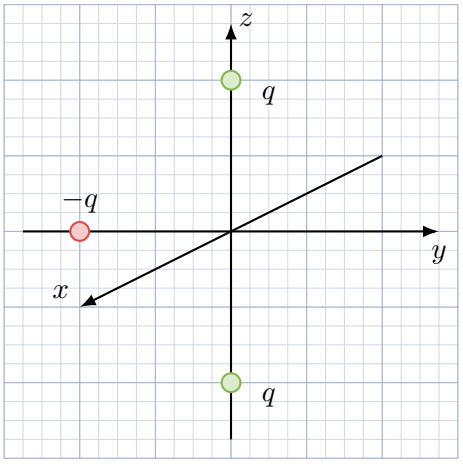
(a) $\lambda(z) = \lambda_0 \cos(\pi z/2a)$

(b) $\lambda(z) = \lambda_0 \sin(\pi z/a)$

(c) $\lambda(z) = \lambda_0 \cos(\pi z/a)$

Problem 5: Multipole expansion for three point charges

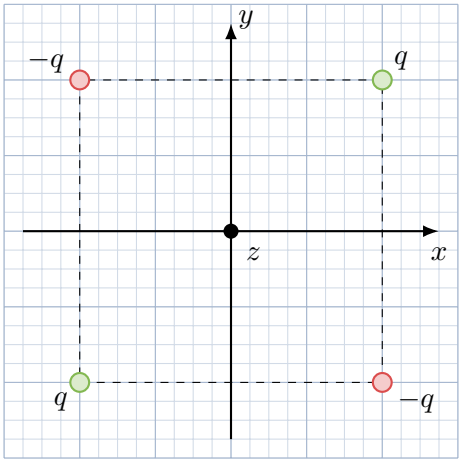
The figure below shows three point charges, each a distance L from the origin:



Find the approximate *electric field* at points far from the charges. Express your answer in spherical coordinates, and include the contributions from the first two non-zero terms in the multipole expansion of the potential.

Problem 6: The quadrupole moment

Find the monopole, dipole, and quadrupole terms in the multipole expansion of the potential for the configuration of charges in the x-y plane shown below. The z-axis in the diagram is pointing out of the page.


y
−q
q
z
x
q
−q