

## Homework 4: Electric Potential

Due: Monday, September 24

### Problem 1: An Impossible Electric Field

One of these vector fields cannot possibly be an electric field. Which one?

(a)  $\vec{E}_1 = \alpha(z^2 + 2xy)\hat{x} + \alpha x^2\hat{y} + 2\alpha zx\hat{z}$

(b)  $\vec{E}_2 = \alpha xy\hat{x} + 2\alpha yz\hat{y} + 3\alpha xz\hat{z}$

Here  $\alpha$  is a constant with appropriate units. For the  $\vec{E}$  which could be an electric field, find the potential at a point with coordinates  $(x, y, z)$ . Use the *origin* as the reference point and find  $V$  by integrating  $d\vec{\ell} \cdot \vec{E}$  along any path that starts at the origin and ends at the point  $(x, y, z)$ . Check that  $-\vec{\nabla}V$  for your answer gives the correct  $\vec{E}$ .

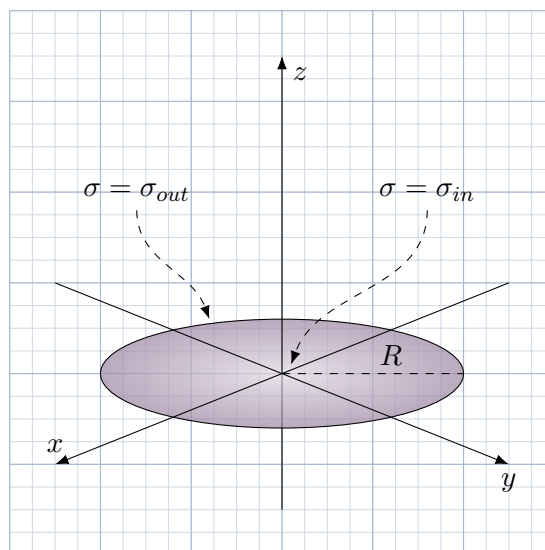
**HINT:** You can use any path between the origin and  $(x, y, z)$  to find  $V$  – they will all give the same result – but to perform the integral you must choose *some* path. Use one that makes the integral easy. For instance, integrate along the  $x$ -axis from  $(0, 0, 0)$  to  $(x, 0, 0)$ , then in the  $y$ -direction from  $(x, 0, 0)$  to  $(x, y, 0)$ , and so on.

### Problem 2: Non-Uniform Surface Charge on a Disk

Find the electric potential at a point above the center of a disk of radius  $R$  with a surface charge density that changes linearly from the center of the disk to its outer edge:

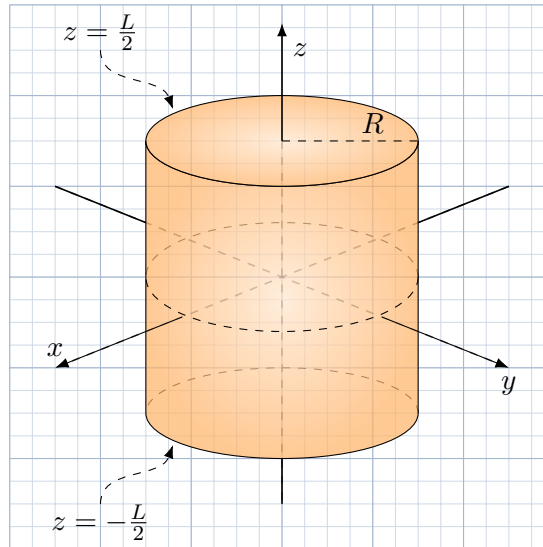
$$\sigma(s) = \sigma_{in} + \frac{s}{R}(\sigma_{out} - \sigma_{in}),$$

where  $\sigma_{in}$  and  $\sigma_{out}$  are constants, and  $s$  is the distance from the center of the disk. To make things easy, assume that the disk sits in the  $x$ - $y$  plane, with the origin at the center of the disk, and use cylindrical polar coordinates. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution.



### Problem 3: Potential due to a Uniform Charged Cylinder

A solid cylinder of length  $L$  and radius  $R$  carries a uniform volume charge density  $\rho$ . Find the potential at a point on the axis of the cylinder, a distance  $z$  from its center. Assume that  $z > L/2$  so the point is outside of the cylinder.



### Problem 4: Potential due to a Solid Uniform Sphere

You already know that the electric potential outside a uniform charged solid sphere of radius  $R$  and total charge  $q$  is the same as a point charge. Calculate the electric potential inside the sphere using the formula

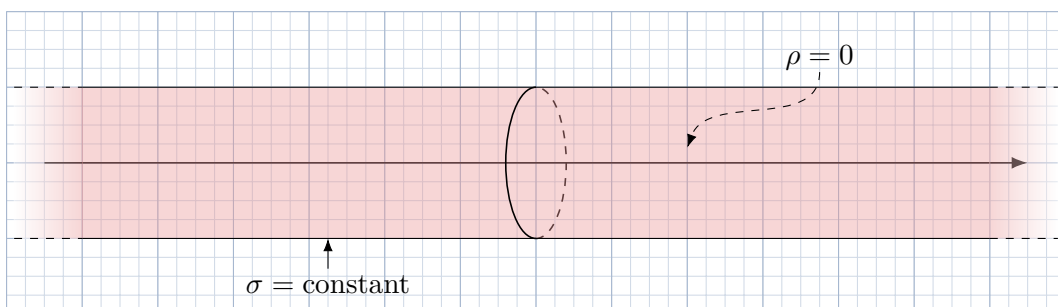
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} d\tau' \frac{\rho(\vec{r}')}{z}$$

You can check your answer by taking the  $R_i \rightarrow 0$  limit of the electric potential for a spherical shell of inner radius  $R_i$  and outer radius  $R$ , which we will work out in class. Finally, use the gradient of your answer to find the electric field inside and outside the sphere.

### Problem 5: Gauss' Law for a Long, Hollow Cylinder

Use Gauss' law to find the electric field inside and outside a long, hollow, cylindrical tube of radius  $R$  which carries a uniform surface charge  $\sigma$ . Check that your result agrees with the expected discontinuity in the component of the electric field normal to the surface:

$$\left( \vec{E}_{out} - \vec{E}_{in} \right) \Big|_{\text{surface}} = \frac{\sigma}{\epsilon_0} \hat{n}$$



### Problem 6: Potential due to a Charged Semicircle

A semicircle of radius  $R$  and uniform line charge  $\lambda$  sits in the  $x$ - $y$  plane. The equation describing the semicircle is

$$x^2 + y^2 = R^2 \quad \text{with} \quad x \geq 0 ,$$

so that the center of the full circle sits at the origin. Calculate the potential at a point on the  $z$  axis using the integral

$$V = \frac{1}{4\pi\epsilon_0} \int d\ell' \frac{\lambda(\vec{r}')}{z} .$$

(This is a pretty simple calculation — if it looks complicated you should double-check your expressions for  $\vec{r}$ ,  $\vec{r}'$ ,  $\vec{z}$ , and  $|\vec{z}|$ .) The electric field at a point on the  $z$  axis should have  $x$  and  $z$  components. Can you determine *both* the  $x$  and  $z$  components at  $(0, 0, z)$  using your expression for the potential? Why or why not?