Problem 1: An Impossible Electric Field
One of these vector fields cannot possibly be an electric field. Which one?

(a) \( \vec{E} = \alpha y z \hat{x} + \alpha (x x - 4 z y) \hat{y} + \alpha (x y - 2 y^2) \hat{z} \).

(b) \( \vec{E} = -3 \alpha x z \hat{x} + \alpha x y \hat{y} + 2 \alpha y z \hat{z} \).

Here \( \alpha \) is a constant with appropriate units. For the \( \vec{E} \) which could be an electric field, find the potential at a point with coordinates \((x, y, z)\). Use the origin as the reference point and find \( V \) by integrating \( d\vec{l} \cdot \vec{E} \) along any path that starts at the origin and ends at the point \((x, y, z)\). Check that \( -\nabla V \) for your answer gives the correct \( \vec{E} \).

HINT: You can use any path between the origin and \((x, y, z)\) to find \( V \) – they will all give the same result – but to perform the integral you must choose some path. Use one that makes the integral easy. For instance, you could integrate along the \( x \)-axis from \((0, 0, 0)\) to \((x, 0, 0)\), then in the \( y \)-direction from \((x, 0, 0)\) to \((x, y, 0)\), and so on. Or you could draw a straight line from the origin to the point and integrate along that: \((x', y', t') = (x t, y t, z t)\) with \(0 \leq t \leq 1\). In that case, \( d\vec{l}' = x dt \hat{x} + y dt \hat{y} + z dt \hat{z} \), which you would dot into \( \vec{E}(x t, y t, z t) \).

Problem 2: Non-Uniform Surface Charge on a Disk
Find the electric potential at a point above the center of a disk of radius \( R \) with a surface charge density that changes linearly from \( \sigma_{in} \) at the center of the disk \( \sigma_{out} \) at its outer edge:

\[ \sigma(s) = \sigma_{in} + \frac{s}{R} \left( \sigma_{out} - \sigma_{in} \right), \]

where \( \sigma_{in} \) and \( \sigma_{out} \) are constants, and \( s \) is the distance from the center of the disk. To make things easy, assume that the disk sits in the \( x \)-\( y \) plane, with the origin at the center of the disk, and use cylindrical polar coordinates. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution.
Problem 3: Potential due to a Uniform Charged Cylinder
A solid cylinder of length $L$ and radius $R$ carries a uniform volume charge density $\rho_0$. Find the potential at a point on the axis of the cylinder, a distance $z$ from its center. Assume that $z > L/2$ so the point is outside of the cylinder.

Problem 4: Potential due to a Solid Uniform Sphere
You already know that the electric potential outside a uniform charged solid sphere of radius $R$ and total charge $Q$ is the same as a point charge. Calculate the electric potential inside a uniformly charged solid sphere using the formula

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int d\tau' \frac{\rho_0}{r},$$

where $\rho_0 = Q/(\frac{4}{3} \pi R^3)$ is the constant charge density. You can check your answer by taking the $R_i \to 0$ limit of the electric potential for a spherical shell of inner radius $R_i$ and outer radius $R$, which we worked out in class. Finally, use the gradient of your answer to find the electric field inside the sphere.

**Hint:** The set-up here is just like the spherical shell example from class. Make sure your $z$-axis is in the direction of $\vec{r}$, so that $\vec{r} = r \hat{z}$. Set up the integrand the same way, and remember that, since $0 \leq r \leq R$, the integral over $r'$ will involve regions where $0 \leq r' < r$ and regions where $r < r' \leq R$. In the former, $\sqrt{(r-r')^2} = r - r'$. In the latter, it’s $\sqrt{(r-r')^2} = r' - r$.

Problem 5: Gauss’ Law for a Long, Hollow Cylinder
Use Gauss’ law to find the electric field inside and outside a long, hollow, cylindrical tube of radius $R$ which carries a uniform surface charge density $\sigma$. Check that your result agrees with the expected discontinuity in the component of the electric field normal to the surface:

$$\vec{E}_{out}(s = R) - \vec{E}_{in}(s = R) = \frac{\sigma}{\epsilon_0} \hat{n}$$
\( \rho = 0 \)

\( \sigma = \text{constant} \)