

Homework 12: The Poynting Vector, Electromagnetic Waves

Due Friday, December 7

Problem 1: Power in a coaxial cable

A long coaxial cable consists of an inner cylinder of radius R_i and an outer cylinder of radius R_o . There is a current I flowing along the surface of the inner cylinder, and a current $-I$ (i.e., the same amount of current in the opposite direction) flowing along the surface of the outer cylinder.

- Use Ampère's law to determine the magnetic field in the region $R_i < s < R_o$ between the cylinders.
- Think of the inner current as a line charge moving at constant velocity: $I = \lambda v$. Use Gauss' law to determine the electric field in the region between the cylinders.
(Remember, $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$ even when the charges are moving, so Gauss' law works the same as before.)
- Calculate the Poynting vector and use it to determine the energy per unit time transported down (along the direction of) the cable.
- Since you know the electric field you can find the potential difference $\Delta V = V_i - V_o$ by integrating $d\vec{l} \cdot \vec{E}$ along a line from the inner cylinder ($s = R_i$) to the outer cylinder ($s = R_o$). Use this to show that the power you computed in the last part is $P = I \Delta V$.

Problem 2: A monochromatic plane wave

A monochromatic plane wave with amplitude E_0 and angular frequency ω is moving in the direction that points from the origin $(0, 0, 0)$ to $(3, -4, 5)$. The polarization of the wave lies in the x - z plane. Determine the components of the (real) electric and magnetic fields for this plane wave (assume that the phase shift of the wave is zero), and calculate the Poynting vector \vec{S} .

Problem 3: A monochromatic spherical wave

Many sources of electromagnetic waves – stars and light bulbs, for example – radiate in all directions. A simple example of the electric field for a monochromatic electromagnetic wave produced by a spherical source is

$$\vec{E}(r, \theta, \phi, t) = A \frac{\sin \theta}{r} \left(\cos(kr - \omega t) - \frac{1}{kr} \sin(kr - \omega t) \right) \hat{\phi}, \quad (1)$$

where A is a constant and $k = \omega/c$.

- Use one of Maxwell's equations to determine the associated magnetic field when there are no sources present (i.e, with $\rho = 0$ and $\vec{J} = 0$). Show that the three remaining equations are all satisfied by these fields.
- Calculate the Poynting vector \vec{S} for this wave, then calculate the *intensity vector* $\vec{I} = \langle \vec{S} \rangle$ by averaging the Poynting vector over a full period $T = 2\pi/\omega$:

$$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T dt \vec{S} \quad (2)$$

- Finally, integrate $\vec{I} \cdot d\vec{a}$ over the surface of a sphere to determine the total power of the wave. This is the average rate at which the spherical source is radiating energy.