

Homework 11: The Vector Potential, Magnetization

Due Friday, November 20

Problem 1: A rotating charged sphere

The vector potential for a spherical shell with uniform surface charge density σ , spinning with constant angular velocity $\vec{\omega}$, is derived in Example 5.11 of *Griffiths*. Repeat this derivation for a spinning solid sphere with uniform volume charge density ρ and no surface charge density. Use the vector potential to determine the magnetic field both inside and outside the sphere.

Problem 2: An infinite solenoid

In class we used Ampère's law to find the magnetic field inside and outside a very long cylindrical solenoid with radius R . Now use Stoke's theorem

$$\int_S d\vec{a} \cdot \vec{B} = \int_S d\vec{a} \cdot (\vec{\nabla} \times \vec{A}) = \oint_{\mathcal{P}} d\vec{\ell} \cdot \vec{A}$$

to find the vector potential both inside and outside the solenoid. Clearly indicate the various surfaces and loops that you use. (Why not just use our integral formula for \vec{A} ? Check what happens when you try to compute \vec{A} that way – it won't work! So why bother calculating \vec{A} if we already know \vec{B} from Ampère's law? Because the vector potential is more than just a convenient way of calculating \vec{B} ; it has physical importance of its own. One example is the [Aharonov-Bohm effect](#).)

Problem 3: Circular loop of wire

A circular loop of wire with radius R lies in the x - y plane, centered at the origin, and carries a current I (running counterclockwise, as seen from above).

- What is the magnetic dipole moment for the loop?
- What is the approximate vector potential at points far ($r \gg R$) from the loop?
- What is the approximate magnetic field at points far from the loop? (Just work out the curl of your answer to the last part!)
- Evaluate your answer for part (c) at a point on the z -axis, and compare it to the $z \gg R$ behavior of the exact expression for the magnetic field on the z -axis that we calculated in class using the Biot-Savart law.

Problem 4: A magnetized cylinder

A very long cylinder with radius R has a “frozen-in” magnetization

$$\vec{M} = k s^2 \hat{\phi},$$

where k is a constant and s is the distance to the cylinder's axis (the z -axis). Find the magnetic field inside and outside the cylinder using the following two methods:

- Find the bound currents associated with the magnetization, then use Ampère's law to determine the magnetic field.
- Determine \vec{H} (keep in mind that there are *no* free currents in this problem), and then use the relationship between \vec{H} , \vec{B} , and \vec{M} to find the magnetic field.