

## Homework 11: The Vector Potential, Magnetization

Due Friday, November 30

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### Problem 1: A rotating charged sphere

The vector potential for a spherical shell with uniform surface charge density  $\sigma$ , spinning with constant angular velocity  $\vec{\omega}$ , is derived in Example 5.11 of *Griffiths*. Repeat this derivation for a spinning solid sphere with uniform volume charge density  $\rho$  and no surface charge density. Use the vector potential to determine the magnetic field both inside and outside the sphere.

### Problem 2: An infinite solenoid

Use Ampère's law to find the magnetic field inside and outside a very long cylindrical solenoid with radius  $R$ . Now use Stoke's theorem

$$\int_S d\vec{a} \cdot \vec{B} = \int_S d\vec{a} \cdot (\vec{\nabla} \times \vec{A}) = \oint_{\mathcal{P}} d\vec{\ell} \cdot \vec{A}$$

to find the vector potential both inside and outside the solenoid. Clearly indicate the various surfaces and loops that you use. (Why not just use our integral formula for  $\vec{A}$ ? Check what happens when you try to compute  $\vec{A}$  that way – it won't work! So why bother calculating  $\vec{A}$  if we already know  $\vec{B}$  from Ampère's law? Because the vector potential is more than just a convenient way of calculating  $\vec{B}$ ; it has physical importance of its own. One example is the [Aharonov-Bohm effect](#).)

### Problem 3: Circular loop of wire

A circular loop of wire with radius  $R$  lies in the  $x$ - $y$  plane, centered at the origin, and carries a current  $I$  (running counterclockwise, as seen from above).

- What is the magnetic dipole moment for the loop?
- What is the approximate vector potential at points far ( $r \gg R$ ) from the loop?
- What is the approximate magnetic field at points far from the loop?
- Evaluate your answer for part (c) at a point on the  $z$ -axis, and compare it to the  $z \gg R$  behavior of the exact expression for the magnetic field on the  $z$ -axis that we calculated in class using the Biot-Savart law.

### Problem 4: Multipole expansion for part of a steady current

In class we worked out the vector potential for a straight segment of current running from  $z = -L$  to  $z = L$ . (Remember: this only makes sense if we think of the straight segment as part of a steady current. We have to interpret the vector potential we found as that segment's contribution to the vector potential for the whole current.) Calculate the first five terms ( $n = 0, 1, 2, 3, 4$ ) in the multipole expansion of  $\vec{A}$  for this straight current. This is not a closed loop of current, so you cannot assume that the monopole term in the multipole expansion is zero.

**Problem 5: A magnetized cylinder**

A very long cylinder with radius  $R$  has a “frozen-in” magnetization

$$\vec{M} = k s^2 \hat{\phi},$$

where  $k$  is a constant and  $s$  is the distance to the cylinder’s axis (the  $z$ -axis). Find the magnetic field inside and outside the cylinder using the following two methods:

- (a) Find the bound currents associated with the magnetization, then use Ampère’s law to determine the magnetic field.
- (b) Determine  $\vec{H}$  (keep in mind that there are *no* free currents in this problem), and then use the relationship between  $\vec{H}$ ,  $\vec{B}$ , and  $\vec{M}$  to find the magnetic field.