Homework 11: The Vector Potential, Magnetization
Due Monday, November 25

Problem 1: A rotating charged sphere

The vector potential for a spherical shell with uniform surface charge density σ, spinning with constant angular velocity $\vec{\omega}$, is derived in Example 5.11 of *Griffiths*. Repeat this derivation for a spinning solid sphere with uniform volume charge density ρ and no surface charge density. Use the vector potential to determine the magnetic field both inside and outside the sphere.

Problem 2: An infinite solenoid

In class we used Ampère's law to find the magnetic field inside and outside a very long cylindrical solenoid with radius R. Now use Stoke's theorem

$$\int_{\mathcal{S}} d\vec{a} \cdot \vec{B} = \int_{\mathcal{S}} d\vec{a} \cdot (\vec{\nabla} \times \vec{A}) = \oint_{\mathcal{P}} d\vec{\ell} \cdot \vec{A}$$

to find the vector potential both inside and outside the solenoid. Clearly indicate the various surfaces and loops that you use.

Comment: Why not just use our integral formula for $\vec{A}$? Check what happens when you try to compute $\vec{A}$ that way – it won't work! So why bother calculating $\vec{A}$ if we already know $\vec{B}$ from Ampère's law? Because the vector potential is more than just a convenient way of calculating $\vec{B}$; it has physical importance of its own. One example is the Aharonov-Bohm effect.

Problem 3: Circular loop of wire

A circular loop of wire with radius R lies in the x-y plane, centered at the origin, and carries a current I (running counterclockwise, as seen from above).

(a) What is the magnetic dipole moment for the loop?

(b) What is the approximate vector potential at points far ($r \gg R$) from the loop?

(c) What is the approximate magnetic field at points far from the loop? (Just work out the curl of your answer to the last part!)

(d) Evaluate your answer for part (c) at a point on the z-axis, and compare it to the $z \gg R$ behavior of the exact expression for the magnetic field on the z-axis that we calculated in class using the Biot-Savart law.

Problem 4: A magnetized cylinder

A very long cylinder with radius R has a "frozen-in" magnetization

$$\vec{M} = k\, s^2\, \hat{\phi}\ ,$$

where k is a constant and s is the distance to the cylinder's axis (the z-axis). Find the magnetic field inside and outside the cylinder using the following two methods:

(a) Find the bound currents associated with the magnetization, then use Ampère's law to determine the magnetic field.

(b) Determine $\vec{H}$, keeping in mind that there are *no* free currents in this problem, and then use the relationship between $\vec{H}$, $\vec{B}$, and $\vec{M}$ to find the magnetic field.