Homework 10: Magnetostatics, Biot-Savart, and Ampère's Law

Due Monday, November 9

Problem 1: Semicircle carrying a steady current

Part of a steady current I flows through a semi-circular segment of wire sitting in the x-y plane. The semi-circle is described by

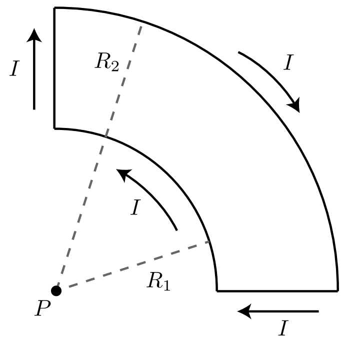
$$x^2 + y^2 = R^2 \quad \text{with} \quad y \geq 0$$

Seen from above ($z > 0$), the current is moving clockwise around the semicircle. Find the contribution to the magnetic field at a point on the z axis due to this part of the current. (If you work in cylindrical coordinates remember to pay close attention to integrals containing unit vectors like $\hat{s}$ or $\hat{\phi}$. To avoid confusion you may want to express them in terms of Cartesian unit vectors before evaluating any integrals.)

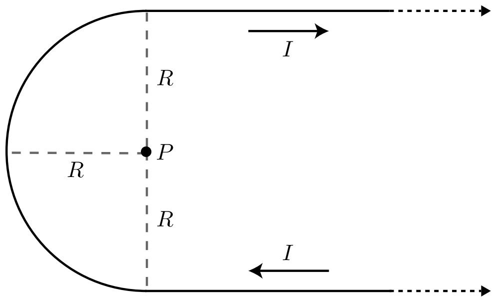
Problem 2: Magnetic field for two current configurations

Use the Biot-Savart law and/or the results of previous problems or examples from class to find the magnetic field at the point P for the two steady current configurations shown below. In both cases the point P is in the plane of the current.

(a)



(b)



Problem 3: Cylinder with volume and surface currents

A long, straight cylinder of radius R carries a uniform current density $\vec{J} = J\,\hat{z}$ on its interior, and a surface current $\vec{K} = -K\,\hat{z}$ on its exterior. Use Ampère's law to find the magnetic field both inside and outside the cylinder. You may treat the cylinder as if it were infinitely long.

Problem 4: Magnetic field in a coaxial cable

A very long coaxial cable consists of an inner conductor and an outer conductor that carry current in opposite directions. The inner conductor is a cylinder ($0 \leq s \leq R_1$) with uniform current density $\vec{J}_i = J\,\hat{z}$, while the outer conductor is a cylindrical shell ($R_2 \leq s \leq R_3$) with uniform current density $\vec{J}_o = -J\,\hat{z}$. The region between the inner and outer conductors ($R_1 < s < R_2$) is empty. Use Ampère's law to find the magnetic field in the regions (i) $s < R_1$, (ii) $R_1 < s < R_2$, (iii) $R_2 < s < R_3$, and (iv) $s > R_3$.

Careful: In problem 3 the magnitudes of $\vec{J}$ and $\vec{K}$ are arbitrary. And in problem 4, there are equal and opposite current densities $\vec{J}_o = -\vec{J}_i$ but the radii R_1, R_2, and R_3 are arbitrary. So don't assume the net current is zero in either problem!