Homework 10: Magnetostatics, Biot-Savart, and Ampère's Law

Due Monday, November 11

Problem 1: Semicircle carrying a steady current

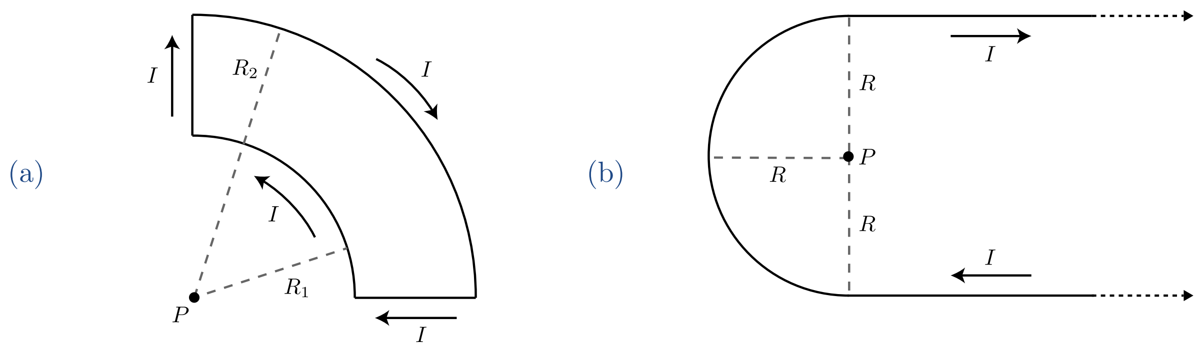
Part of a steady current I flows through a semi-circular segment of wire sitting in the x-y plane. The semi-circle is described by

$$x^2 + y^2 = R^2 \quad \text{with} \quad y \geq 0$$

Seen from above ($z > 0$), the current is moving clockwise around the semicircle. Find the contribution to the magnetic field at a point on the z axis due to this part of the current. (This is similar to an example we did in class. If you work in cylindrical coordinates remember to pay close attention to integrals containing unit vectors like $\hat{s}$ or $\hat{\phi}$. To avoid confusion you may want to express them in terms of Cartesian unit vectors before evaluating any integrals.)

Problem 2: Magnetic field for two current configurations

Use the Biot-Savart law and/or the results of previous problems or examples from class to find the magnetic field at the point P for the two steady current configurations shown below. In both cases the point P is in the plane of the current. You can think of $\hat{z}$ as the direction pointing out of the page.



Problem 3: Cylinder with volume and surface currents

A long, straight cylinder of radius R carries a uniform current density $\vec{J} = J\,\hat{z}$ on its interior, and a surface current $\vec{K} = -K\,\hat{z}$ on its exterior. Use Ampère's law to find the magnetic field both inside and outside the cylinder. You may treat the cylinder as if it were infinitely long.

Problem 4: Magnetic field in a coaxial cable

A very long coaxial cable consists of an inner conductor and an outer conductor that carry current in opposite directions. The inner conductor is a cylinder ($0 \leq s \leq R_1$) with uniform current density $\vec{J}_i = J\,\hat{z}$, while the outer conductor is a cylindrical shell ($R_2 \leq s \leq R_3$) with uniform current density $\vec{J}_o = -J\,\hat{z}$. The region between the inner and outer conductors ($R_1 < s < R_2$) is empty. Use Ampère's law to find the magnetic field in the regions (i) $s < R_1$, (ii) $R_1 < s < R_2$, (iii) $R_2 < s < R_3$, and (iv) $s > R_3$.

Careful: In problem 3 the magnitudes of $\vec{J}$ and $\vec{K}$ are arbitrary. And in problem 4, there are equal and opposite current densities $\vec{J}_o = -\vec{J}_i$ but the radii R_1, R_2, and R_3 are arbitrary. So don't assume the net current is zero in either problem!