

## Homework 10: Magnetostatics, Biot-Savart, and Ampère's Law

Due Monday, November 12

### Problem 1: Semicircle carrying a steady current

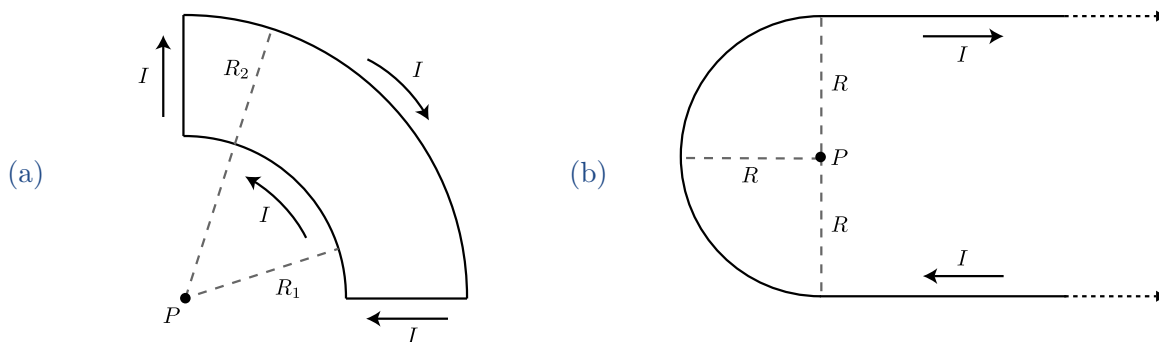
A steady current  $I$  flows through a semi-circular segment of wire sitting in the  $x$ - $y$  plane. The semi-circle is described by

$$x^2 + y^2 = R^2 \quad \text{with } y \geq 0$$

Find the magnetic field at a point on the  $z$  axis due to this current. (If you work in cylindrical coordinates remember to pay close attention to integrals containing unit vectors like  $\hat{s}$  or  $\hat{\phi}$ . To avoid confusion you may want to express them in terms of Cartesian unit vectors before evaluating any integrals.)

### Problem 2: Magnetic field for two current configurations

Use the Biot-Savart law and/or the results of previous problems or examples from class to find the magnetic field at the point  $P$  for the two steady current configurations shown below. In both cases the point  $P$  is in the plane of the current.



### Problem 3: Cylinder with volume and surface currents

A long, straight cylinder of radius  $R$  carries a uniform current density  $\vec{J} = J \hat{z}$  on its interior, and a surface current  $\vec{K} = -K \hat{z}$  on its exterior. Use Ampère's law to find the magnetic field both inside and outside the cylinder. You may treat the cylinder as if it were infinitely long.

### Problem 4: Magnetic field in a coaxial cable

A very long coaxial cable consists of an inner conductor and an outer conductor that carry current in opposite directions. The inner conductor is a cylinder ( $0 \leq s \leq R_1$ ) with uniform current density  $\vec{J}_i = J \hat{z}$ , while the outer conductor is a cylindrical shell ( $R_2 \leq s \leq R_3$ ) with uniform current density  $\vec{J}_o = -J \hat{z}$ . The region between the inner and outer conductors ( $R_1 < s < R_2$ ) is empty. Use Ampère's law to find the magnetic field in the regions (i)  $s < R_1$ , (ii)  $R_1 < s < R_2$ , (iii)  $R_2 < s < R_3$ , and (iv)  $s > R_3$ .

**CAREFUL:** In problem 3 I don't say anything about the relative size of  $J$  and  $K$ . And in problem 4, there are equal and opposite current densities  $\vec{J}_o = -\vec{J}_i$  but the radii  $R_1$ ,  $R_2$ , and  $R_3$  are arbitrary. So don't assume the net current is zero in either problem!