This assignment is due at our second class, on September 1st.

This is the first homework assignment for Physics 351. It is a review to make sure you are up to speed on a few topics covered in your multivariable calc and Physics 301 (Math Methods) classes. We will spend the first week reviewing some more complicated topics, but you should already know the material covered in this assignment when class starts. There are two important rules for this assignment:

- You may not use MATHEMATICA (or Maple, Matlab, Wolfram Alpha, etc) on any of these problems. Everything must be done by hand.
- You should do these problems on your own. On future assignments I’ll encourage you to collaborate with classmates, but I want you to do this assignment on your own so you can assess whether or not you are familiar with the material.

Think very carefully about the second point. This is a review of skills that are absolutely essential for this class. You really need to make sure you are comfortable with this stuff before we get started, and the only way to do that is to work these problems out yourself. If you have questions you can email me at rmcnees@luc.edu or stop by my office (Cudahy 314).

Problem 1: Div, Grad, and Curl Identities
Let \( f(x, y, z) \) and \( g(x, y, z) \) be scalar functions, and let \( \vec{A}(x, y, z) \) and \( \vec{B}(x, y, z) \) be vector functions. Prove the following identities involving divergence, gradient, and curl.

\[
\nabla (f + g) = \nabla f + \nabla g \\
\n\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B} \\
\n\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B} \\
\n\nabla (fg) = g \nabla f + f \nabla g \\
\n\nabla \cdot (f \vec{A}) = f \nabla \cdot \vec{A} + \vec{A} \cdot \nabla f \\
\n\n\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}) \\
\n\n\nabla \times (f \vec{A}) = \nabla f \times \vec{A} + f \nabla \times \vec{A}
\]

In this case, “prove” means you should show enough steps so that someone looking over your work can tell that you convinced yourself that each one of these identities is correct.

Problem 2: Divergence and Curl

Compute the divergence and curl of the following vectors:

(a) \( \vec{r} = x \hat{x} + y \hat{y} + z \hat{z} \).

(b) The unit vector \( \hat{r} = \frac{\vec{r}}{|\vec{r}|} \).
(c) $\vec{V} = z\hat{x} + x\hat{y} + y\hat{z}$

(d) $\vec{w} = -x^2 y\hat{x} + y^2 x\hat{y} + x y z\hat{z}$

**Problem 3: Position Vector**

Suppose $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ is a constant vector, and $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the position vector of a point with coordinates $(x, y, z)$. Evaluate the following:

(a) $\vec{\nabla} \times (\vec{k} \times \vec{r})$

(b) $\vec{\nabla} (\vec{k} \cdot \vec{r})$

**Problem 4: Line Integrals**

Given the vector $\vec{V} = x y\hat{x} + \frac{3}{2} y^2\hat{y}$, evaluate the line integral

$$\int_{(0,0)}^{(3,3)} d\vec{\ell} \cdot \vec{V} \tag{8}$$

from the point $(0,0)$ to the point $(4,8)$, along the following paths:

(a) $y = x^2 - 2x$

(b) Along $y = 0$ from $x = 0$ to $x = 4$, then along $x = 4$ from $y = 0$ to $y = 8$.

(c) Along $x = 0$ from $y = 0$ to $y = 8$, then along $y = 8$ from $x = 0$ to $x = 4$.

In this problem we’re considering paths in the $x$-$y$ plane; there is no $z$ direction to worry about. If you don’t yet have a copy of the textbook, $d\vec{\ell}$ is the notation Griffiths uses for the infinitesimal displacement vector $dx\hat{x} + dy\hat{y}$, so $d\vec{\ell} \cdot \vec{V} = V_x dx + V_y dy$. **Remember:** this is a line integral. Take the time to review line, surface, and volume integrals if you aren’t clear on what each one is and how they are different! There is a set of review notes on line integrals available here.